4. B. P. Gerasimov, T. G. Elizarova, and V. I. Turchaninov, "The method of suppression of network viscosity in the solution of the Navier–Stokes equations," Differents. Uravn., No. 7, 1165-1173 (1984).
5. A.A. Samarskii, "Contemporary applied mathematics and numerical experiment," Kommunist, No. 18, 31-42 (1983).
6. E. V. Shil'nikov, "Calculation of the effect of laser radiation on gaseous carbon," in: Dynamics of a Radiating Gas [in Russian], No. 3, Computing Center, Academy of Sciences of the USSR, Moscow (1980), pp. 25-32.
7. A. A. Samarskii, Theory of Difference Schemes [in Russian], Nauka, Moscow (1977).
8. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press, New York (1966).
9. E. Ya. Litovskii and N. A. Puchkevich, Thermophysical Properties of Refractory Materials [in Russian], Metallurgiya, Moscow (1982).
10. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composite Materials [in Russian], Énergiya, Leningrad (1974).
11. V. A. Petrov, "Optical properties of quartz glasses at high temperatures in the region of their semitransparency," in: Compilations of Thermophysical Properties of Materials, No. 3(17) [in Russian], Institute of High Temperatures, Academy of Sciences of the USSR, Moscow (1979), pp. 29-72.

NUMERICAL-ANALYTICAL MODELING OF HEAT TRANSFER BETWEEN A LAMINAR FLOW AND HIGHLY PERMEABLE ROUGHNESS

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A mathematical model is proposed for the thermal interaction of a laminar flow with a layer of small stationary streamlined obstacles. Numerical and analytical investigations exhibit characteristic zones of the flow.

A concept that has proved useful for the solution of a number of practical problems [1-4] is the notion of highly permeable roughness (HPR), which we interpret in the present study as a plane layer $0 \leqq x < \infty$, $0 \leqq z \leqq h$ randomly filled with stationary streamlined obstacles. We assume for definiteness that the obstacles are nondeformable spheres of diameter d, which is much smaller than the thickness h of the HPR. Their concentration is small enough that hydrodynamic and thermal interaction does not take place between them.

Let an unbounded viscous fluid flow with temperature-independent properties move along the HPR. The intensity of interaction of the flow with the HPR is determined by the local velocity U(z) of the flow relative to the obstacles, the local temperature difference $\Theta - \vartheta$ between the fluid and the obstacles, and the concentration (number density) n of the obstacles per unit volume. Owing to the smallness of the concentration, $\partial p/\partial z = 0$. A mathematical model of the flow produced by the HPR can be written in the form of boundary-layer equations with source terms. The latter have a discontinuity at the line of demarcation between the hindered and external flows, z = h [1, 2]:

$$f=\begin{cases}\rho knU, \\ 0,\end{cases} \quad i=\begin{cases}\alpha n(\Theta-\vartheta)S, & 0\leqslant z\leqslant h, \\ 0, & z>h.\end{cases} \tag{1}$$

The drag coefficient k (m^3/sec) of the obstacles is assumed to be constant. For small Reynolds numbers $Re' = U_\infty d/\nu < 1$, e.g., $k = 3\pi\nu d$ (Stokes' law). Analogously, α = const and does not depend on the flow velocity. The hydrodynamic interaction of a flow with HPR has been investigated in our previous work. Here we turn our attention to the heat-transfer problem.

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We introduce the dimensionless variables

$$\bar{z}=\frac{z}{h},\quad \bar{x}=\frac{x}{\mathrm{Re}\,h},\quad \bar{U}=\frac{U}{U_\infty},\quad \bar{V}=\mathrm{Re}\,\frac{V}{U_\infty},$$
$$\bar{\Theta}=\frac{\Theta-\Theta_0}{\Theta_\infty-\Theta_0},\quad \bar{\vartheta}=\frac{\vartheta-\Theta_0}{\Theta_\infty-\Theta_0}, \tag{2}$$

where $\mathrm{Re} = U_\infty h/\nu$. Then in the HPR layer $0 \leq z \leq 1$ the problem acquires the form (the overbar designating dimensionless symbols is dropped from now on)

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial z}=\frac{\partial^2 U}{\partial z^2}-AU,$$
$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial z}=0,$$
$$U\frac{\partial \Theta}{\partial x}+V\frac{\partial \Theta}{\partial z}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 \Theta}{\partial z^2}-A_t(\Theta-\vartheta), \tag{3}$$
$$z=0\quad U=V=\Theta=0,$$
$$x=0\quad U=\Theta=1.$$

The same boundary-layer equations hold in the free-stream flow outside the HPR, $z > 1$, but without the source terms ($A = A_t = 0$). At a large distance from the HPR ($z \to \infty$) the velocity and temperature of the flow are equal to unity: $U = \Theta = 1$. Problem (3) contains three dimensionless groups:

$$A=\frac{knh^2}{\nu},\quad A_t=\frac{n\alpha S h^2}{c\rho\nu},\quad \mathrm{Pr}=\frac{\nu}{a}. \tag{4}$$

In the variables (2) the boundary conditions for the velocity and temperature become identical. The dimensionless heat flux is conveniently introduced in the form

$$\bar{q}=\frac{qh}{\lambda(\Theta_\infty-\Theta_0)}=\frac{d\bar{\Theta}}{d\bar{z}}.$$

The numerical investigation is based on a two-layer six-point implicit differencing scheme [5]. The thermal regime is computed after solution of the dynamical problem. The physical parameters n, k, α and, hence the dimensionless groups A and A_t, are assumed to be constant.

1. Let the temperature of the obstacles be held constant and equal to the wall temperature: $\vartheta = 0$. Then it is evident from equations (3) that the velocity and temperature profiles will coincide under the condition $A = A_t$, $\mathrm{Pr} = 1$ (Reynolds analogy). The calculated profiles for this case are shown in Fig. 1a. The variation of the profiles with increasing coordinate x mirrors the transformation of the flow by the HPR. The flow is heated uniformly in the initial cross section $x = 0$. As the flow progresses along the HPR and interacts with the obstacles and the wall surface, it loses its thermal energy. For $A = A_t = 0$ (absence of the HPR, $n = 0$) we would have an ordinary thermal boundary layer on a cold surface. The zone of influence of the HPR and the wall (outer boundary layer) expands along the path of the motion. As a result of the exchange of energy between the flow layers, the flow acquires a transverse heat flux (dashed curves in Fig. 1) from the heated outer flow into the depth of the HPR. This process affects only the upper levels of the HPR at small x, but then it penetrates deeper and deeper and in a number of cases reaches the lower level $z = 0$. The influx of thermal energy merely counteracts the temperature drop of the flowing medium at first, but at large x equilibrium sets in between the energy influx and the energy sink created by the obstacles. Evidence of this fact is found in the bulging of the profiles in the interior of the HPR layer and their tendency toward a certain limit.

Thus, the thermal transformation of the flow is determined by three processes: the heat sink at the obstacles, the wall sink, and the replenishment of thermal energy from the outer flow. The pattern of the effects involved here stands out more sharply if the influence of the wall is excluded. Accordingly, we consider another HPR model, in which the layer of obstacles $-h \leq z \leq h$ is suspended in the flow. Then the line $z = 0$ takes the role of a symmetry axis, on which the following boundary conditions hold:

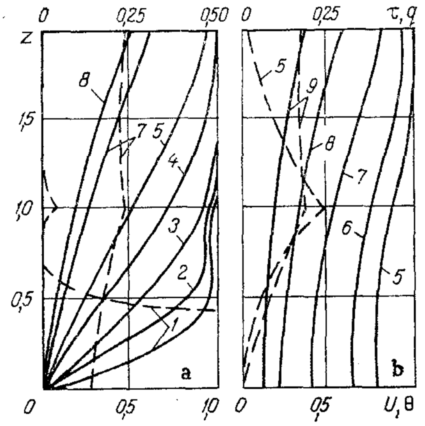

Fig. 1

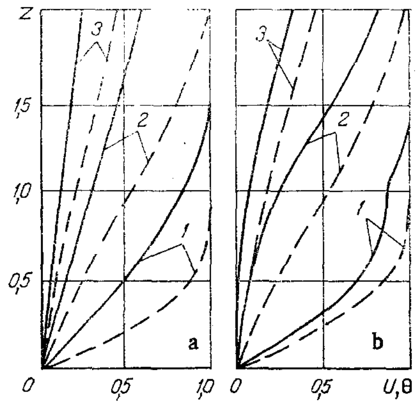

Fig. 2

Fig. 1. Calculated profiles: velocity and temperature (solid curves); friction and heat flux (dashed curves) in the case of the Reynolds analogy $A = A_t$, $Pr = 1$ at various distances from the start of the HPR layer, $A = 1$. a) Presence of a wall at the lower level of the HPR; b) HPR suspended in the flow. 1) $x \cdot 10^2 =$ 1, 2) 2, 3) 5, 4) 10, 5) 20, 6) 40, 7) 80, 8) 160, 9) 320.

Fig. 2. Calculated velocity (dashed curves) and temperature (solid curves) profiles at various distances x from the start of the HPR for $A = 1$ and: $A_t = 1$, $Pr = 0.1$; b) $A_t = 10$, $Pr =$ 1. 1) $x \cdot 10^2$; 2) 20; 3) 160.

$$z = 0 \; \frac{\partial U}{\partial z} = 0, \; V = 0, \; \frac{\partial \Theta}{\partial z} = 0. \tag{5}$$

The calculated profiles for such a HPR are shown in Fig. 1b. The variation of the flow in this case takes place under the action of the transfer of thermal energy by the obstacles and its admission from outside the HPR. The strictly vertical parts of the profiles refer to the regions of the HPR where only a thermal energy sink is observed and the influence of the outer flow is absent [$q(z) = 0$, dashed curves]. The zone unaffected by the influence of the outer flow is situated above and below the axial line $z = 0$ and narrows with increasing x as a result of the boundary layer, which grows downward from the HPR level $z = 1$. Equilibrium between the sink and influx of thermal energy sets in at large x, resulting in stabilization of the profile $\Theta(x, z)$ with respect to the variable x.

The zone in which energy is not admitted [$q(z) = 0$] has been called the "potential core" in the analysis of the hydrodynamic flow pattern. When a wall is present at the lower level of the HPR, the "potential core" narrows more rapidly, because the influence of the upper boundary layer is augmented by that of the wall boundary layer. The flow zone $0 \leq x \leq L$, where the "potential core" exists, is called the initial zone. The zone characterized by stabilization of the profiles in the interior of the HPR is the main zone. It can be assumed with a certain fractional error that the main zone is situated immediately after the initial zone, i.e., in the inverval $L \leq x < \infty$.

The pattern is the same in principle for other values of the parameters of the problem. Figure 2a shows how the temperature profiles vary for different values of Pr. They become separate from the velocity profiles (which are represented by dashed curves), and the Reynolds analogy breaks down. For $Pr < 1$ the outer thermal boundary layer grows more rapidly than the dynamical boundary layer, characterizing the quantity $1/Pr$ as the rate of thermal energy transfer between the flow and the HPR.

The variation of A_t also violates the analogy between the hydrodynamic and thermal processes (Fig. 2b). An analysis of extreme cases exhibits the tendency of the flow pattern to vary as A_t is varied. The case $A_t = 0$ has the physical significance, according to (4), that the heat-transfer coefficient α of the obstacles is equal to zero. With a decrease in A_t, therefore, the curves approach the limit $\Theta = 1$. The flow pattern becomes reminiscent of a thermal boundary at the surface $z = 0$. The difference is only a consequence of the difference in the velocity distributions U(x, z) and V(x, z) in the case of a boundary layer and in the investigated problem, since $A \neq 0$.

The case $A_t \to \infty$ corresponds to a very small specific heat of the flowing fluid: $c \to 0$. Heat transfer to the obstacles therefore causes the temperature to drop to zero. Consequently, an increase in A_t is accompanied by the tendency of the temperature profile in the HPR layer to the limit $\Theta \equiv 0$, $0 \leq z \leq 1$. The limiting flow resembles a thermal boundary layer on the cold surface z = 1. A comparison of Fig. 2b with Fig. 1a confirms these conclusions as to the influence of the parameter A_t. The foregoing discussion characterizes this parameter as the rate of absorption of thermal energy of the flow in the interior of the HPR.

Depending on the relationship of the parameter A, the input rate of thermal energy 1/Pr, and its absorption rate A_t, the situation is possible where the thermal influence of the outer flow is felt only in the upper levels of the HPR, $\ell \leq z \leq 1$, so that the temperature of the flowing medium is equal to zero below the level $z = \ell$. The corresponding analytical case is shown in Fig. 2b. The thermal-energy penetration depth $\ell \approx 0.53$ for it. This has the practical implication that devices with such parameters will be inefficient.

2. Based on our knowledge of the characteristic mechanisms involved in the initial and main flow zones, certain analytical estimates can be obtained for them.

In the "potential core" region of the initial zone, a vertical variation of the flow characteristics does not take place, and $\partial/\partial z = 0$. This assumption enables us to simplify the equations of motion nd energy:

$$U\frac{dU}{dx} = -AU,\ U\frac{d\Theta}{dx} = -A_t\Theta \tag{6}$$

($\vartheta = 0$). The first equation gives the velocity decay law in the potential core inside the HPR: $U = 1 - Ax$ for $0 \leq x \leq L$ and $U = 0$ for $x \geq L$. Here the quantity

$$L = 1/A = \frac{\nu}{knh^2} \tag{7}$$

is the approximate length of the hydrodynamic initial zone. Making use of this fact, from the second equation (6) we obtain the temperature decay law in the flow:

$$\Theta = (1 - Ax)^{A_t/A}, \tag{8}$$

where $A_t/A = \alpha S/c\rho k$.

For $A_t = A$ the lengths of the thermal and dynamical zones coincide and are expressed by Eq. (7). For $A_t > A$ the thermal initial zone is shorter than the hydrodynamic initial zone, and its length is estimated by the formula

$$L_t = (1 - \varepsilon^{A/A_t})/A, \tag{8}$$

where $\varepsilon = 0.05$. For $A_t/A = 5$, e.g., the ratio of the two lengths $L_t/L = 0.45$. Equations (7) and (8) show, in addition, that stabilization of the hydrodynamic and thermal processes sets in more rapidly, the greater the concentration of obstacles in the HPR and the greater its height. The Prandtl number does not exert any influence in the initial zone.

The main zone, in contrast, is typified by independence of the velocity and temperature profiles from the coordinate x. Under this assumption problem (3) acquires the simpler form

$$U'' - AU = 0,\ \Theta'' - A_t \Pr(\Theta - \vartheta) = 0,$$
$$z = 0 \quad U = \Theta = 0. \tag{9}$$

It is seen that the hydrodynamic parameter A and the initial temperature profile do not affect the thermal processes in this part of the flow. These processes are determined by the single dimensionless parameter $\omega^2 = A_t/\Pr = \alpha n S h^2/\lambda$. The hydrodynamics of the stabilized flow has been investigated in [1], and the heat transfer in [2]. The solution of the thermal problem can be written in the form

$$\Theta = \Theta_h \frac{\operatorname{sh}\omega z}{\operatorname{sh}\omega}. \tag{10}$$

Here the flow temperature at the upper level of the HPR $\Theta_h = \Theta(x, 1)$ is an additional parameter, which must be determined from the calculation of the boundary layer over the surface

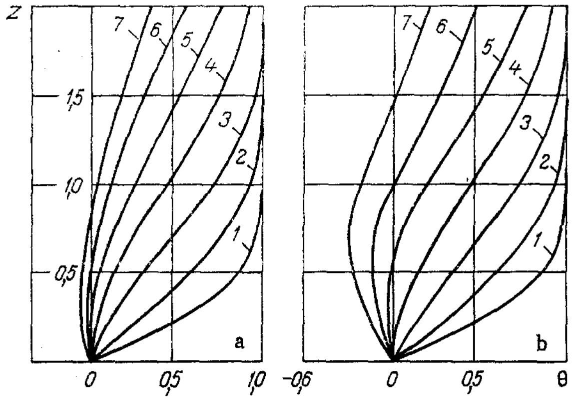

Fig. 3. Calculated velocity and temperature profiles for various thermal regimes of the HPR. a) Different temperatures of the obstacles and the wall; b) regulated heat release from the obstacles, $A = A_t = 1$. 1) $x \cdot 10^2 = 2$; 2) 5, 3) 10, 4) 20, 5) 40, 6) 80, 7) 160.

$z = 1$. Equation (10) can be used to estimate analytically the HPR depth $z = \ell$ to which thermal energy from the outer flow penetrates:

$$l = \frac{1}{\omega} \operatorname{Ar\,sh}(\varepsilon \operatorname{sh} \omega) \approx 1 + \frac{1}{\omega} \ln \varepsilon, \qquad (11)$$

where the quantity $\varepsilon = \Theta(\ell)/\Theta$ and can be taken equal to $\varepsilon = 0.05$. For example, for $\omega = 10$, thermal energy from the outer flow penetrates the interior of the HPR only to the level $\ell = 0.70$, below which $\Theta \equiv 0$.

The foregoing discussion is also valid when the HPR is suspended in the flow, as distinguished by condition (5). Its temperature profile, stabilized in the main zone, has the form

$$\Theta = \Theta_h \frac{\operatorname{ch} \omega z}{\operatorname{ch} \omega}.$$

For large values of ω it differs very little from the profile (10) because, as we infer from (11), the flow temperature is equal to zero in a certain region $0 \leq z \leq \ell$ above the wall, and the solution is not affected by it.

The analytical relations (10) and (11) for the main zone are well corroborated by the numerical calculations. The expressions (7) and (8) for the length of the initial zone are valid only for large values of A and A_t/A, for which the kinetic and thermal energies of the outer flow penetrate the HPR to comparatively shallow depths. On the whole, however, these expressions do not completely characterize the influence of the outer flow on the processes taking place in the interior of the HPR and can therefore only be used as approximations.

3. In a number of problems the temperature of the obstacles differs from the wall temperature. In the dimensionless variables (2) this means that $\vartheta = \text{const} \neq 0$ in equations (3). The calculation of this model is illustrated in Fig. 3a for the case of a wall temperature different from the HPR and free-stream temperatures: $\vartheta = -0.5$. The analysis of the flow pattern, which is considerably more complicated than the case $\vartheta = 0$, can be carried out by the analytical methods proposed for the initial and main zones. In the former, the temperature drop along the flow obeys the law $\Theta = \vartheta + (1 - Ax)^{A_i/A}$. The temperature profile, stabilized in the main zone

$$\Theta = \Theta_h \frac{\operatorname{sh} \omega z}{\operatorname{sh} \omega} + \vartheta \left[(1 - \operatorname{ch} \omega z) - (1 - \operatorname{ch} \omega) \frac{\operatorname{sh} \omega z}{\operatorname{sh} \omega}\right],$$

can have an inflection point. The profile bends toward the limit $\Theta = \vartheta$.

Another interesting case is control of the thermal regime of the HPR in such a way that the quantity of heat admitted to its elements is held constant; in wind tunnel experiments, e.g., the voltage and current passed through the HPR model are set. Then the power i_0 of the distributed heat release (1) is constant, so that the source term in (3) retains a constant value $A_t(\Theta - \vartheta) = i_0$, while the temperature of the obstacles is related one-to-one with the local temperature of the flowing medium by the algebraic expression

$$\vartheta = \Theta - i_0/A_t,$$

i.e., varies along the height of the HPR. In the initial zone the free-stream temperature decays according to the law

$$\Theta = 1 - \frac{i_0}{A} \ln(1 - Ax).$$

A parabolic temperature profile is formed in the main zone:

$$\Theta = \Theta_h z + \frac{1}{2} i \Pr z(z-1), \tag{12}$$

wherein the parameter $\Theta_h = \Theta(1)$ must be determined by analyzing the outer boundary layer. This type of flow pattern is shown in Fig. 3b for a special analytical solution. The heat sink created by the obstacles in this problem is so strong that the temperature profiles acquire a pronounced concavity. The profile 6, which is already close to the stabilized profile, is very accurately described by Eq. (12).

NOTATION

x, z, longitudinal and transverse coordinates; U, V, longitudinal and transverse flow velocities; ρ, density; μ, ν, dynamic and kinematic viscosities; c, specific heat of the obstacles; Θ, flow temperature; ϑ, temperature of the obstacles; λ, a, thermal conductivity and thermal diffusivity; α, heat-transfer coefficient, $J/m^2 \cdot K \cdot sec$; τ, viscous friction; $q = -\lambda \partial\Theta/\partial z$, vertical heat flux, $J/m^2 \cdot sec$; k, drag coefficient of obstacles; h, thickness of layer of obstacles (HPR); d, $S = \pi d^2$, diameter and surface area of individual obstacle; n, concentration of obstacles; f, i, source functions of distributed force and heat; $Re = U_\infty h/\nu$, Reynolds number; $Re' = U_\infty d/\nu$, local Reynolds number; Pr, Prandtl number; A, ω, dimensionless parameters; ℓ, thermal-energy penetration depth; L, length of initial zone; i, power of distributed heat release. Indices: ∞, free-stream ($z \to \infty$) values of velocity and temperature; 0, values of temperature at wall; h, values of temperature at upper boundary of the layer of obstacles; t, heat-transfer process.

LITERATURE CITED

1. E. A. Gaev and I. K. Nikitin, "Hydrodynamics of flow in the presence of highly permeable roughness: laminar regime," in: Fluid Mechanics, Republic Joint Collection [in Russian], No. 45, Naukova Dumka, Kiev (1982), pp. 65-73.
2. E. A. Gaev, "Fundamental laws of heat transfer between a flow and a special type of roughness," in: Heat Transfer and the Thermophysical Properties of Materials [in Russian], Naukova Dumka, Kiev (1984), pp. 90-95.
3. A. S. Dubov, L. P. Bykova, and S. V. Marunich, Turbulence in a Vegetative Cover [in Russian], Gidrometeoizdat, Leningrad (1978).
4. R. S. Bortkovskii, Heat and Moisture Transfer in the Atmosphere and Ocean during a Storm [in Russian], Gidrometeoizdat, Leningrad (1983).
5. I. Yu. Brailovskaya and L. A. Chudov, "Solution of boundary-player equations by a differencing method," in: Computational Methods and Programming [in Russian], No. 1, Moscow State Univ. (1962), pp. 167-182.